Three-dimensional stagnation point flow of a second grade fluid towards a moving plate

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Abstract

The problem dealing with the steady three-dimensional flow of a second grade fluid near the stagnation point of an infinite plate moving parallel to itself with constant velocity has been investigated. By using the appropriate transformations for the velocity components and temperature, the basic equations governing flow and heat transfer have been reduced to a set of ordinary differential equations. These equations have been solved approximately subject to the relevant boundary conditions by employing a numerical technique. The effect of a nondimensional elastic parameter on the velocity components, wall shear stress, temperature and heat transfer has been examined carefully.

Keywords: Stagnation point; Viscoelasticity; Second grade fluid

1. Introduction

Most problems involving two- and three-dimensional stagnation point flow have similarity solutions in the sense that the number of independent variables is reduced by one or more. These similarity solutions may be derived using the group-theoretic method. The analysis of such flows is very important in both theory and practice. From a theoretical point of view, flows of this type are fundamental in fluid mechanics and forced convective heat transfer. From a practical point of view, these flows have applications in many manufacturing processes in industry such as the boundary layer along material handling conveyers, the aerodynamic extrusion of plastic sheet, and the cooling of an infinite metallic plate in a cooling bath.

The classical two-dimensional Hiemenz [1] and axisymmetric Homann [2] stagnation point flows describe situations where fluid impinges normally onto a flat surface and spreads out bidirectionally or radially along the surface, away from a single stagnation point. Both two-dimensional and axisymmetric flows were extended
to three dimensions by Howarth [3] and Davey [4]. Two-dimensional oblique stagnation flow was solved by Stuart [5] and later by Tamada [6] and Dorrepaal [7].

Authors like Stuart [8], Rott [9], and Glauert [10] analyzed the two-dimensional stagnation point flow against a plate that is oscillating in its own plane. Yang [11] investigated the two-dimensional unsteady stagnation point flow towards a plate. Yang’s work was extended by Williams [12] for the case of axisymmetric flow and then Cheng et al. [13] for the case of three-dimensional flow. The three-dimensional stagnation point flow on a moving plate was considered by Wang [14] and Liby [15]. Wang [16] studied the unsteady oblique stagnation point flow. Weidman and Mahalingam [17] solved the problem of axisymmetric stagnation point flow impinging on a flat plate oscillating in its own plane with suction and blowing by reduction of the Navier–Stokes equations to a set of coupled ordinary differential equations and subsequent numerical integration. Laminar incompressible mixed convection boundary layer flow with large injection rates at the stagnation point of a three-dimensional body was examined by Eswara and Nath [18].

All the above investigations are, however, confined to flows of Newtonian fluids. In recent years, it has generally been recognized that in industrial applications non-Newtonian fluids are more appropriate than Newtonian fluids. For instance, in certain polymer processing applications, one deals with the flow of a non-Newtonian fluid over a moving surface. That non-Newtonian fluids are finding increasing application in industry has given impetus to many researchers. Srivastava [19] obtained an approximate solution for an axisymmetric flow of a Reiner–Rivlin fluid near a stagnation point adopting the Karman–Pohlhausen method used for the study of boundary layer equations in Newtonian fluids. Maiti [20] re-examined the same flow problem by replacing Reiner–Rivlin fluid by power-law fluid. For second-order Rivlin–Ericksen fluid, Rajeswari and Rathna [21] studied the two-dimensional and axisymmetric flows near a stagnation point by using an extension of the Karman–Pohlhausen technique. The Prandtl boundary layer theory was extended by Beard and Walters [22] for an idealized elastico-viscous fluid, more specifically such a fluid is called a Walter’s B’ fluid, and then by Sarpkaya and Rainey [23] for a second-order viscoelastic fluid. They obtained the approximate solution valid for sufficiently small values of the elastic parameter by employing a perturbation procedure, using the coefficient that multiplies the highest order term in the equation as the perturbation parameter, thereby lowering the order of the equation. Soundalgekar and Vighnesam [24] used the same perturbation scheme in order to obtain a solution to the heat transfer problem related to the two-dimensional stagnation point flow of Walter’s B’ fluid. Garg and Rajagopal [25] considered the two-dimensional stagnation point flow of thermodynamically compatible second-order fluids, where only the velocity field was studied. They obtained solutions valid for all values of an elastic parameter by using an additional boundary condition at infinity. The heat transfer aspect of this problem was investigated Massoudi and Ramezan [26] and Garg [27]. Dorrepaal et al. [28] examined the behaviour of a viscoelastic fluid impinging on a flat rigid wall at an arbitrary angle of incidence. Labropulu et al. [29] studied the orthogonal and oblique flows of a second grade fluid impinging on a wall with suction or blowing. Ariel [30] examined the generalized three-dimensional stagnation point flow of a Walter’s B’ fluid against a stationary flat plate by using the transformations proposed by Howarth [3] for the velocity components. He has demonstrated on the basis of his exact numerical solutions that the solutions can be obtained only up to some critical value of the elastic parameter, and that for values less than this critical value dual solutions exist. In his subsequent study [31], he investigated the laminar, steady stagnation point flow of a Walter’s B’ fluid towards a moving plate by considering both the cases of two-dimensional and axisymmetric flow. Seshadri et al. [32] studied the unsteady three-dimensional stagnation point flow of a viscoelastic fluid of second grade. The two-dimensional stagnation point flow of a second grade fluid was investigated by Ariel [33]. In this study, it is shown that without augmenting the boundary conditions at infinity it is possible to obtain a numerical solution of the problem for all values of the dimensionless viscoelastic fluid parameter. Recently, Mahapatra and Gupta [34] have made an analysis of the steady two-dimensional stagnation point flow of an incompressible viscoelastic fluid of short memory (obeying Walter’s B’ model) over a flat deformable surface when the surface is stretched in its own plane with a velocity proportional to the distance from the stagnation point. In another study Labropulu and Chinichian [35] have analyzed the unsteady stagnation point flow of the Walter’s B’ fluid impinging obliquely on a flat plate oscillating in its own plane.

In the present paper, our concern is to investigate the steady three-dimensional flow of a second grade fluid towards a stagnation point at an infinite plate moving parallel to itself with constant velocity. The problem
reduces to the solution of a set of ordinary differential equations in which for each equation the order of the highest derivative is one more than the number of available boundary conditions. To overcome this requirement of additional conditions we have augmented the boundary conditions at infinity as in [25]. The algorithm proposed in [36] has been used to compute the velocity and temperature distributions.

2. Formulation of the problem

There are many fluids whose behaviour cannot be described by the classical Navier–Stokes model. The inadequacy of the theory of Newtonian fluids in predicting the behaviour of some fluids, especially those with high molecular weight, leads to the developments of non-Newtonian fluid mechanics. Among the many models that have been used to describe the non-Newtonian behaviour exhibited by certain fluids, the second order fluid model has received special attention. In this model, the constitutive equation is given by the following relation for incompressible fluids (cf. Truesdell and Noll [37])

\[ T = -pI + \mu A_1 + x_1 A_2 + x_2 A_1^2, \]  

where

\[ A_1 = \nabla v + (\nabla v)^T, \quad A_2 = \frac{D}{Dt} A_1 + A_1 \cdot \nabla v + (\nabla v)^T \cdot A_1. \]

In the above equations, the spherical stress \(-pI\) is due to the constraint of incompressibility, \(\mu\) is the viscosity, \(x_1\) and \(x_2\) are material moduli usually referred to as the normal stress coefficient, \(D/Dt\) is the material time derivative, \(v\) denotes the velocity field, \(V\) is the gradient operator, and the tensors \(A_1\) and \(A_2\) are the first two Rivlin–Ericksen tensors (cf. Rivlin and Ericksen [38]).

If the fluid modeled by Eq. (1) is to be compatible with thermodynamics in the sense that all motions of the fluid meet the Clasius–Duhem inequality and the assumption that the specific Helmholtz free energy of the fluid is a minimum in equilibrium, then (cf. Dunn and Fosdick [39])

\[ \mu \geq 0, \quad x_1 \geq 0, \quad x_1 + x_2 = 0. \]  

The fluids characterized by above restrictions are called the second grade fluids in the literature. On the other hand, the model equation (1) is called a second order fluid model if it is not required to be compatible with thermodynamics [40,41]. The sign of the coefficient \(x_1\) has been a subject of much controversy. Dunn and Fosdick [39] demonstrated that a second grade fluid exhibits acceptable stability characteristics. Later, Fosdick and Rajagopal [42] showed that the fluid exhibited anomalous behaviour not to be expected of any fluid of rheological interest if \(x_1 < 0\) and \(x_1 + x_2 \neq 0\). It is very important to bear in mind that solutions to steady flow problems can be found when \(x_1 < 0\). However, all these flows are not stable [43]. Several such solutions corresponding to the case \(x_1 < 0\) are presented in the recent book of Truesdell and Rajagopal [44]. As a result, in any event, the results established for the case \(x_1 > 0\) have more value than the solution for \(x_1 < 0\). In this study, we shall assume that the model under consideration meets Eq. (3), and is compatible with present literature.

We consider the orthogonal steady three dimensional stagnation point flow against an infinite plate at \(z = 0\) moving with constant velocity \(U\) in the \(x\)-direction. A non-Newtonian fluid flowing in the direction negative \(z\)-axis approaches a moving plate at \(z = 0\), and divides into streams proceeding away from the stagnation point at the origin.

For three-dimensional flow let the fluid far from the plate, as \(z\) tends to infinity, be driven by the potential flow

\[ u_\infty = ax, \quad v_\infty = ay, \quad w_\infty = -2az, \]  

where \((u,v,w)\) are velocities in the Cartesian \((x,y,z)\) directions, and \(a\) is a physical constant with dimensions of \(T^{-1}\), depending on the velocity in potential motion. Then, from the Euler equation the pressure distribution will be

\[ p = p_0 - \frac{\rho a^2}{2} (x^2 + y^2 + 4z^2), \]
where $\rho$ is the density and $p_0$ is the pressure at the stagnation point. Since the velocity field given in Eq. (4) does not satisfy the no-slip conditions at the plate, it is not an acceptable solution of the equations of viscous flow. The problem is to obtain a solution that satisfies the no-slip boundary conditions and agrees with the outer solution far from the stagnation point. Following Wang [14] we seek a similarity solution compatible with the continuity equation through the variables

$$u = Uf(\eta) + axh'(\eta), \quad v = ayh'(\eta), \quad w = -2\sqrt{\alpha}h(\eta),$$

(6)

where $v = \mu/\rho$ is called the kinematic viscosity, $\eta = \sqrt{a/v_z}$ and the prime denotes the derivative with respect to $\eta$. It is important to note that the function $f(\eta)$ represents velocity profile due to the translation of the plate at $z = 0$.

The boundary conditions for the velocity field are

$$u(0) = U, \quad v(0) = 0, \quad w(0) = 0,$$

$$\lim_{z\to\infty} u \to u_\infty = ax, \quad \lim_{z\to\infty} v \to v_\infty = ay, \quad \lim_{z\to\infty} w \to w_\infty = -2az.$$

(7)

The assumptions made in this analysis are as follows: (a) the flow is steady and laminar, (b) the fluid is incompressible, (c) the body forces are negligible, (d) all the physical fluid properties are constant. Under the above stated assumptions, the basic equations of the problem are as follows:

Continuity equation:

$$\nabla \cdot v = 0.$$  

(8)

Equations of motions (cf. [39]):

$$\rho \left( \frac{1}{2} \nabla |v|^2 + w \times v \right) = -\nabla p + \mu \nabla^2 v + \alpha_1 \left[ \nabla^2 w \times v + \nabla \left( v \cdot \nabla v + \frac{1}{4} |A_1|^2 \right) \right].$$

(9)

Energy equation:

$$\frac{\rho D e}{D t} = \mathbf{T} \cdot \nabla v - \nabla \cdot \mathbf{q} + \rho r.$$  

(10)

From [39], we know that

$$\mathbf{T} \cdot \nabla v = \frac{\mu}{2} |A_1|^2 + \frac{\alpha_1}{4} \frac{D}{D t} |A_1|^2.$$  

(11)

In the above equations $e$ is the specific internal energy, $\mathbf{q}$ the heat flux vector, $r$ the radiant heating, $w = \nabla \times v$, and the norm $| |$ denotes the usual norm for vectors and the trace norm for tensors. Note that $\alpha_2$ is eliminated by using restriction (3)_3.

Substituting Eq. (6) into the equations of motion (9) and using the conditions of integrability, we get

$$h''' + 2hh'' - h^2 + 1 + S(2h'h'' - 2hh'V - h'^2) = 0,$$

$$f'' + 2hf' - h'f + S(-2hf'' + h'f'' - h''f' + h''f) = 0,$$

(12)

(13)

where $S$ is the dimensionless measure of the viscoelastic fluid parameter $\alpha_1$ given by

$$S = \frac{\alpha_1 a}{\mu}.$$  

(14)

The boundary conditions (7) are re-written as

$$h(0) = 0, \quad h'(0) = 0, \quad f(0) = 1, \quad \lim_{\eta \to \infty} h'(\eta) \to 1, \quad \lim_{\eta \to \infty} f(\eta) \to 0.$$  

(15)

The terms in Eqs. (12) and (13) having the $S$ factor represent the viscoelastic character of the fluid. It is noticed that the system of equations characterizing the flow has an order of seven, but there are only five boundary conditions. To obtain a solution we need two extra boundary conditions. For flows that take place in unbounded domains, it has been shown by Garg and Rajagopal [25,45] that the boundary conditions can be augmented by the fact the solution has to be bounded or has a certain smoothness at infinity. Following Garg and Rajagopal [25] two extra boundary conditions for the problem under consideration can be written as
\[ \eta \to \infty : \lim_{\eta \to \infty} h''(\eta) \to 0, \quad \lim_{\eta \to \infty} f'(\eta) \to 0. \tag{16} \]

It may be noted that the conditions in Eq. (16) are the asymptotic boundary conditions on \( h \) and \( f \). The first condition was also used in [25] with the aim of obtaining a numerical solution. The numerical solution of Eqs. (12) and (13) subject to the related boundary conditions (15) and (16) will be described later.

Having computed the velocity field, substituting the results back into equations of motion and then integrating it can be shown that the general expression for the pressure distribution is

\[ p(x, y, \eta) = -\frac{D}{2} [4va(h' + h^2 - 2Shh' - 3Sh^2) + a^2(x^2 + y^2)(1 - 2Sh'^2) - 2USf'(Uf' + 2axh'')] + P_0, \tag{17} \]

where \( P_0 \) is a constant reference pressure. In the absence of \( S \), Eqs. (12), (13) and (17) are the same as those obtained by Wang [14].

It is also of interest to determine the effect of viscoelastic parameter \( S \) on the shear stress on the plate in the \( x \)-direction. From the constitutive equation of a second grade fluid, we obtain

\[ \tau_w = \frac{1}{\mu U \sqrt{a/v}} T_{zz} \bigg|_{z=0} = f'(0) + \left( \frac{ax}{U} + S \right) h''(0). \tag{18} \]

Next, we introduce a temperature field of the form

\[ T = T_\infty + (T_w - T_\infty) \theta(\eta), \tag{19} \]

where \( T_\infty \) is the temperature of the fluid at infinity and \( T_w \) is the temperature of the plate. We shall assume that the heat flux vector \( q \) satisfies Fourier’s law with a constant thermal conductivity \( k \), i.e.,

\[ q = -k \nabla T. \tag{20} \]

The energy equation for the problem under consideration, with the assumption of negligible dissipative effects and radiant heating, can be represented by the following equation:

\[ \theta'' + 2Pr \theta' = 0, \tag{21} \]

where \( Pr \) is the Prandtl number.

Eq. (21) is to be solved subject to the boundary conditions

\[ \theta(0) = 1, \quad \lim_{\eta \to \infty} \theta(\eta) \to 0. \tag{22} \]

In order to solve Eq. (21), the function \( h \) is first determined from Eq. (12) and it can then be solved numerically.

The heat transfer rate per unit area on the plate can be written by Fourier’s law as follows:

\[ q_w = -k \left( \frac{\partial T}{\partial z} \right)_{z=0} = -k \sqrt{\frac{a}{v}} (T_w - T_\infty) \theta'(0). \tag{23} \]

3. Numerical results and discussion

Eqs. (12), (13) and (21) under the relevant conditions given in Eqs. (15), (16) and (22) were solved numerically using the Matlab solver singular boundary value problem (SBVP) designed for the solution of two-point boundary value problems. The code is based on collocation at either equidistant or Gaussian collocation points. An error estimate for the global error of the approximate solution is also provided. This estimate provides the basis for an adaptive mesh selection strategy. The mesh points are automatically modified with the aim to equidistributing the global error. A detailed description is given in [36].

The algorithm mentioned above was used to compute the flow for various values of nondimensional elastic parameter \( S \) in the interval \( 0 \leq S \leq 0.6 \). The differential equations (12), (13) and (21) were integrated from \( \eta = 0 \) to \( \eta = \eta_\infty \), where \( \eta_\infty \) is a sufficiently large number. In practice, setting \( \eta_\infty \) as low as 8 yields satisfactory
accuracy for the present calculations. As a test of accuracy of the solution, it may be noted that at \( \eta = 0 \), Eqs. (12) and (13) reduce to

\[
\begin{align*}
\frac{d^2}{dh^2}(0) - S \frac{d}{dh}(0) + 1 &= 0, \\
\frac{d^3}{dh^3}(0) + S(\frac{d^2}{dh^2}(0) - \frac{d}{dh}(0)f''(0)) &= 0.
\end{align*}
\]

(24) (25)

It was found that for all solutions computed for \( 0 \leq S \leq 0.6 \), the left hand sides of (24) and (25) were less than \( 10^{-8} \). Also, \( f''(\eta_{\infty}) \) and \( h''(\eta_{\infty}) \) were found to be less than \( 10^{-3} \) in all cases. Moreover, results for \( S = 0.01 \) were very close to those for \( S = 0 \) in which case Eqs. (12) and (13) are the same as those obtained by Wang [14].

Table 1 provides the missing (unspecified) initial conditions at the initial point (\( \eta = 0 \)).

In a viscoelastic fluid, such as a second grade fluid, boundary layers can occur due to a variety of reasons and boundary layers with multiple decks are possible, with the confinement of different quantities in the various decks. In the flow of a second grade fluid, normal stress as well as shear stress field differ from the corresponding stress fields of a Newtonian fluid. Thus, if a boundary layer is to exist in flows of such fluids, it is necessary that not only the ratio of the inertial forces to the forces due to the tangential stresses be large, but also the ratio of the inertial forces to the forces due to the normal stresses should be large. In the case of a Newtonian fluid, the condition of the inertial forces being much larger than the shear forces implies the Reynolds number \( Re \) should be very large. For a second grade fluid, the ratio of inertial forces to the forces due to the normal stresses is given by the ratio of the Reynolds number \( Re \) to the nondimensional parameter, namely \( We \), related to the normal stress modulus \( \alpha_1 \). For a Newtonian fluid, the boundary layer theory is valid when \( Re \gg 1 \). In the case of a second grade fluid, boundary layer formation is observed when \( Re \gg 1 \) and \( Re/We \gg 1 \) as pointed out in [46]. In the flow of a second grade fluid, when the above conditions are satisfied, in addition to the classical viscous boundary layer, an elastic boundary layer within which the normal stresses are large appears. If \( We \ll 1 \), then the normal stress effects would not be important and we retrieve the classical viscous boundary layer. We refer the reader to the works of Rajagopal et al. [46] and Rajagopal [47] regarding the detailed discussion on this subject. In this paper we are only concerned with inertial boundary layers. We wish to emphasize here that Eqs. (12) and (13) are one order higher than the Navier Stokes equations due to the viscoelasticity of the fluid and thus would require additional boundary conditions. Since we are in a situation of semi-infinite region, such boundary conditions may be assumed in the form of asymptotic conditions at infinity. This is reflected in our boundary conditions (16).

The predictions based on the foregoing analysis are displayed graphically for various values of nondimensional elastic parameter \( S \) in Figs. 1–4. Figs. 1–3 show the velocity profiles corresponding to the \( x, y \) and \( z \) directions, respectively. We observe from these figures that the main effect of elasticity of the fluid on the three-dimensional stagnation point flow against a moving flat plate is to increase in velocity component in the \( x \)-direction, whereas it is to decrease the velocity components in the \( y \) and \( z \) directions. The temperature profiles are presented in Fig. 4. It is apparent from Fig. 4 that the temperature profiles slightly increase with the increase in elasticity of the fluid. Again from Fig. 4, we arrive at the conclusion that the thermal boundary layer thickness becomes small for the increase in Prandtl number, as expected. It is also noted that the temperature distribution is independent of plate translation. The values of shear stress on the plate in the \( x \)-direction (\( \tau_{\alpha} \)) are tabulated in Table 2 for different values of the parameters \( ax/U \) and \( S \). We conclude from Table 2

Table 1: Missing initial conditions

<table>
<thead>
<tr>
<th>( S )</th>
<th>( f''(0) )</th>
<th>( f''(0) )</th>
<th>( h''(0) )</th>
<th>( h''(0) )</th>
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Fig. 1. The velocity component in the \(x\)-direction.

Fig. 2. The velocity component in the \(y\)-direction.

Fig. 3. The velocity component in the \(z\)-direction.
that an increase in elastic parameter $S$ leads to a reduction in the value of wall shear stress $\tau_w$. Table 3 illustrates the effect of elastic parameter $S$ on the heat transfer rate per unit area on a plate for a selection of values of the Prandtl number. From Table 3, we note that with an increase in elastic parameter $S$, the heat loss per unit area from the plate decreases. This change in heat transfer is more pronounced for a large Prandtl number.

### Table 2
Values of wall shear stress

<table>
<thead>
<tr>
<th>$S$</th>
<th>$\tau_w$</th>
<th>$ax/U = 0.01$</th>
<th>$ax/U = 0.1$</th>
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### Table 3
Values of heat transfer parameter $-\theta'(0)$

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<th>$S$</th>
<th>$-\theta'(0)$</th>
<th>$Pr = 0.1$</th>
<th>$Pr = 1$</th>
<th>$Pr = 20$</th>
<th>$Pr = 50$</th>
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</table>
4. Conclusions

In the present paper we have considered the stagnation point flow of a particular class of viscoelastic fluids, known as second grade fluid, towards a moving plate. By means of similarity transformations, the governing equations have been reduced to a set of ordinary differential equations. The resulting two point boundary value problems have the feature that the order of the system of differential equations exceeds the number of available boundary conditions. Nevertheless we have obtained the exact numerical solution by augmenting the boundary conditions at infinity as in [25]. We have integrated numerically the relevant differential equations using the Matlab solver SBVP designed by Auzinger et al. [36]. Numerical calculations have been carried out for various values given to the nondimensional parameters and the significant contributions of the elastic parameter $S$ to the velocity components, temperature distribution, shear stress and heat transfer have been pointed out.

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